CHAPTER 6--ENERGY

QUESTION SOLUTIONS

6.1) A net force accelerates a body. If you multiply that force by the distance over which it is applied, what will that quantity tell you?

Solution: When a net force is applied over a distance, the product of the force component in the direction of motion times the distance over which the force acts gives you what is called the *work* done by the force over the distance. Its significance is that it is related to the amount of *velocity change* that occurs due to the application of that force over that distance. The relationship, called *the work energy theorem* (a bit of a misnomer, but that's life), is $W_{net} = .5mv_f^2 - .5mv_o^2$, where the $.5mv^2$ term is called *kinetic energy*. In short, the product alluded to in the question (net force times distance) tells you by how much the body's kinetic energy *changes* due to the application of the force.

6.2) A net force F stops a car in distance d. In terms of F, how much force must be applied to stop the car in the same distance if its velocity is tripled?

Solution: Using the definition of work (i.e., $\mathbf{F}\cdot\mathbf{d}$) and building upon the comments in *Problem 1*, we can write $\mathbf{F}\cdot\mathbf{d} = .5m(\Delta v)^2$. In this case, the final velocity is zero. Clearly, the left hand side of this equation is proportional to the velocity *squared.* With $\mathbf{d}$ constant and $\mathbf{F}$ proportional to v^2, increasing v by a factor of three increases $\mathbf{F}$ by a factor of *three squared*, or nine.

6.3) An object of mass m moving with speed v comes to rest over a given distance d due to the effects of friction. What do you know about the average frictional force involved (i.e., how large must it have been)?

Solution: Using the idea of the work/energy theorem, assuming that the only force acting to stop the object is friction and remembering that the final velocity is zero, we can write $\mathbf{f}_k\cdot\mathbf{d} = 0 - .5mv_o^2$. The work done by friction will be negative (the angle between $\mathbf{f}$ and $\mathbf{d}$ is 180^o, so the cosine in the *dot product* is -1) and the frictional force will equal $.5mv_o^2/d$.

6.4) Two masses, m and $2m$, both freefall from rest. Ignoring friction, which has the greater speed after falling a given distance? Which has more work done to it by gravity over that distance? Is there something to explain here? If so, do so.

Solution: As you probably know, all objects accelerate gravitationally at the same rate in a vacuum (i.e., without frictional effects). The answer to the first question, therefore, is that both will have the *same speed* after falling a given distance. As for the work done, the gravitational force on the mass m will be mg while the gravitational force on the mass $2m$ will be $2mg$. But the distances traveled are the same for both masses, so gravity will do twice as much work on the $2m$ mass as it does on the m mass. Is this weird? Not really. Being more massive (read this *more inert*), it should take more energy to get the larger mass moving. As it happens, the kinetic energy change is going to be twice as big for the $2m$ mass, so there must be twice the work done. Note: If you look at the expression that relates force and velocity--the work/energy theorem--this situation yields a work quantity of mgd and a change of kinetic energy quantity of $.5mv_f^2 - 0$. Putting it together yields $mgd = .5mv_f^2$. Notice that the masses cancel (this would also be the case if you wrote out this relationship for the $2m$ situation). In other words, assuming the body starts from rest, the only variable that affects the final velocity is the *square root of the fall distance*. In a frictionless situation, the mass has nothing to do with velocity . . . which is exactly what we acknowledged at the start of this little discussion.

6.5) A car slows from 40 m/s to 20 m/s, then from 20 m/s to 0 m/s. In which instance (if any) was more energy pulled out of the system? Reversing the question, going from zero to 20 m/s requires more, the same, or less energy than is required to go from 20 m/s to 40 m/s? Explain.

Solution: The work, hence energy, required to accelerate a car from zero to 20 m/s will be less than the work required to accelerate the car from 20 m/s to 40 m/s. Why? Because according to the work/energy theorem, work is not linearly related to velocity, it is related to *the square* of the velocity. Specifically, for the zero to 20 m/s situation, the work/energy expression yields $W_{net} = .5m(20)^2 - .5m(0)^2 = 200m$ *joules*. For the 20 to 40 m/s situation, the work/energy expression yields $W_{net} = .5m(40)^2 - .5m(20)^2 = 600m$ *joules*. Clearly it takes more energy to accelerate a vehicle that is already moving than it does to accelerate one that starts from rest. Likewise, one must take *more energy* out of a fast moving car to slow it down by 20 m/s than it does to slow a slow moving car down by 20 m/s.

6.6) A force is applied to an object initially at rest. The force acts over a distance d taking the object up to a speed v.

a.) If the force had been halved but the distance remained the same, how would the final velocity have changed (if at all)?

Solution: Halving the force while keeping the distance constant would have halved the work being done (remember, work is $\mathbf{F} \cdot \mathbf{d}$). Because work is related to velocity squared, one would expect the velocity to decrease . . . but not by half. If the original work had been W_{orig}, doing the math for the new situation yields $W_{orig}/2 = .5mv_{new}^2$. We know that $W_{orig} = .5mv^2$, so we can substitute that in for W_{orig} in the first expression re-writing it as $(.5mv^2)/2 = .5mv_{new}^2$. Canceling appropriately leaves us with $(1/2)^{1/2}v = v_{new}$, or $v_{new} = .707v$.

b.) If, instead, the distance had been halved with the force remaining unchanged, how would the final velocity have changed (if at all)?

Solution: As was the case in *Part a*, the work quantity would still halve and v_{new} would still equal $.707v$.

6.7) What is the ONE AND ONLY thing potential energy functions do for you?

Solution: A potential energy function is a derived quantity that is attached to a specific conservative force. The one and only thing it allows you to determine is the amount of work that that force does as a body moves from one point to another in the force's field. To get that work quantity, all you have to do is evaluate the potential energy function at the beginning and ending points, then take minus the difference between the two. That is, $W_{done\ by\ cons.\ force\ fld.} = -\Delta U$, where U is the symbol most often used to denote potential energy. Note that in the *conservation of energy* expression, you have what appears to be single potential energy quantities, not differences. If you look back at the way the *conservation of energy equation* was derived, though, you will find that in the beginning of the derivation those potential energy quantities were introduced to determine the amount of work conservative forces in the system did as the body went from the initial to the final point. That is, they were introduced as *potential energy differences*.

6.8) An ideal spring is compressed a distance x. How much more force would be required to compress it a distance $2x$? How much more energy would be required to execute this compression?

Solution: The relationship between force and displacement of an ideal spring is linear, so if you double the displacement, you have to double the force. The potential energy function for an ideal spring, on the other hand, is a function of the *square* of the displacement ($U = .5kx^2$, where k is the spring constant and x is measured from the spring's equilibrium position). As such, doubling the displacement will require *four times* the energy.

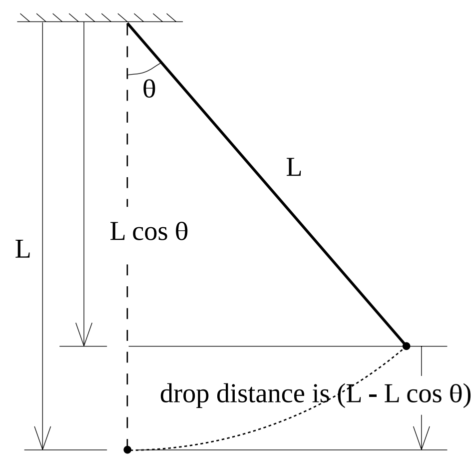
6.9) A mass moving with speed v strikes an ideal spring, compressing the spring a distance x before coming to rest. In terms of v, how fast would the mass have to be moving to compress the spring a distance $2x$?

Solution: You are converting kinetic energy, a function that depends on the velocity squared, into *spring potential energy*, a function that depends on the displacement squared. In other words, as both are squared quantities, the relationship between the displacement and velocity here is linear (they are both of the same order--both squared in the defining expression) and depressing the spring a distance $2x$ would require a velocity of $2v$.

6.10) A simple pendulum (a mass attached to a string) is pulled back to an angle θ and released. Ignore friction.

a.) If the mass is doubled, what will happen to the velocity at the bottom of the arc?

Solution: This is back to the *what falls faster in a vacuum, a cannonball or a feather* question. Because the gravitational force motivating the mass to move has to overcome the inertia of the body, doubling the mass doubles the gravitational force but also doubles the inertia . . . and you get no net difference between the two situations. Put another way, the velocity of the "falling" bob is not dependent upon the size of the bob's mass.



b.) If the length of the pendulum arm is doubled, how will the velocity at the bottom of the arc change?

Solution: Doubling the length of the pendulum arm will increase the fall distance of the bob as it swings down. How much is the increase? The sketch shows how much the bob "falls" in general. Noting that the bob starts from rest and assuming that the gravitational potential energy is zero at the bottom of the arc, the work/energy theorem (or the conservation of energy expression--both will yield the same relationship) yields $W_{grav} = -\Delta U = -[0 - mg(L - L\cos\theta)] = .5mv^2 - 0$, or $mg(L - L\cos\theta) = .5mv^2$. This could also be written as $mgL(1 - \cos\theta) = .5mv^2$ In other words, the arm length L is proportional to the *square* of the velocity. Doubling the length of the arm, therefore, means the velocity increases by $(2)^{1/2}$.

c.) Is there any acceleration at the bottom of the arc? If so, how much and in what direction?

Solution: This is a bit of a tricky question. When the bob is at the bottom of its arc, there are no forces acting in the horizontal. With no horizontal acceleration acting at that point, it is the bob's motion alone that allows it to continue on through the bottom of the arc and out again. In the vertical, it is not uncommon for people to think that because there is no motion in that direction, there must be no acceleration. Unfortunately, that is not true as the body is following a *curved path*. That is to say, there must be some non-zero force and, hence, acceleration to motivate it out of straight-line motion. Indeed, tension and gravity exist in opposition at that point, but the two don't add to zero. They combine to act as a centripetal force equal to $T - mg$. In most cases, we don't know what the tension T is, but there is a clever way around that problem. Specifically, the centripetal force IS EQUAL TO mv^2/r (remember, the centripetal acceleration is equal to v^2/r, where r is the radius of the motion). Therefore, if we can determine v at the bottom of the arc, we can determine the *center seeking* acceleration. Fortunately for us, the *conservation of energy* expression coupled with the gravitational potential energy function gives us an easy way to determine the velocity of a body that has "fallen" in a gravitational field.

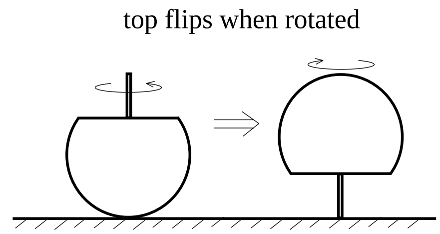
d.) How much work does tension do as the bob moves from the initial point to the bottom of the arc?

Solution: In this case, tension is always at right angles to the motion. That means it does no work on the system at any point in time.

e.) How much work does gravity do as the bob moves from the initial point to the bottom of the arc?

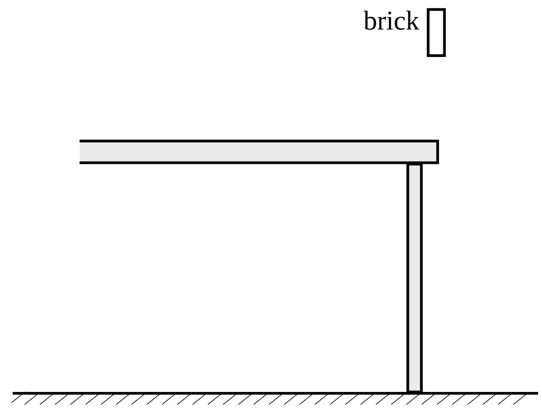
Solution: Trying to do this using $\boldsymbol{F}\cdot\boldsymbol{d}$ would be a huge pain in the arse. Why? Because the angle between the direction of motion and gravity would be constantly changing (in fact, if you tried to do this calculation you'd have to use the *integral form* of the work equation). Fortunately for you, all you have to do is determine the vertical distance the bob falls from its initial point to the bottom of the arc (for those of you who are shaky on such things, that distance is shown in the sketch attached to *Part b*), then use the gravitational potential energy function and the fact that $W_{grav} = -\Delta U_{grav}$ (remember, the only way you will ever use a potential energy function is to determine how much work its force field does as the body goes from one point to another in the field).

6.11) There is a toy on the market--a top--that, when spun, flips itself over (see sketch). What is the top really doing as it moves from the one state to the other state?



Solution: In general, systems in nature tend to migrate to states of least energy. By flipping over, the top's center of mass lowers thereby decreasing its gravitational potential energy.

6.12) A brick is held above the edge of a table. Suzy Q looks at the brick, deduces that if it were to fall it would land ON the table, and calculates the brick's gravitational potential energy with that in mind. In doing so, she comes up with a number N_1. Big Jack, who happens to have terrible eyesight and has left his glasses at home, looks at the brick and decides that if it falls, it will land on the ground. He keeps that in mind as he calculates the brick's gravitational potential energy coming up with a number N_2. Which potential energy quantity is correct? Explain.



Solution: In most cases, the zero potential energy level associated with a force function is defined as the position at which the force is zero. In Newton's *GENERAL* expression for gravity, for instance, that place is at infinity (you haven't run into this situation, yet). For a spring, it's at the spring's equilibrium position. When you are dealing with gravity *close to the surface of the earth*, the problem is that there *is* no place it's zero--to a good approximation, gravity is a *constant* near the earth's surface. That means two things. First, the near-earth gravitational potential energy function is linear (i.e., $U = mgy$ . . . no big deal). And second, there is no preferred position at which the gravitational potential energy must be zero. ANY POINT WILL DO. There's nothing wrong with this. Remember, a potential energy function evaluated at a particular point means nothing. It isn't until you subtract it from the evaluation of the function at some other point that it takes on significance. What significance? Minus the difference in the evaluations tells you how much work the force field does as a body goes from the one point to the other. As *differences* are all that matter, and as the function is linear for near-earth gravity, you can define the zero level to be anywhere you want and the function will still be able to do for you what it is supposed to do. In short, taking either the tabletop or floor to be the zero potential energy level will work just fine as long as you stick with your chosen zero level throughout the problem.

6.13) For a spring system, it is very obvious when there is no potential energy wrapped up in the position of the spring. For a gravitational situation near the surface of the earth, that isn't the case. What is the telltale *difference* between the two situations?

Solution: As outlined in *Question 12*, gravitational force near the earth is constant and, hence, has no preferred zero level. As such, gravitational potential energy can be zeroed anywhere. A spring applies a force that is related to how much the spring is elongated or compressed (the relationship is $F = -kx$, where k is the spring's *spring constant* and x is the spring's displacement). At equilibrium, there is no displacement and, hence, no force, so the spring's potential energy is zero there.

6.14) Is it possible for:

a.) Potential energy to be negative? If yes, give an everyday example.
Solution: If, in *Question 12*, you had made the zero level of the gravitational potential energy function be at the tabletop, any object below that point would have had a negative potential energy (think about the potential energy expression for near earth gravity--it is mgy, where the y variable denotes the vertical distance between the point and the zero level--if you are below the zero level, y will be negative). Also, there are potential energy functions that, due to their defined zero point, are inherently negative (when far from the earth, the gravitational function is an example). That's OK. Remember, it is minus the *difference* in the evaluation of the potential energy function that tells you how much work is being done by the field as you go from one point to another. As long as that is true, it doesn't matter what the sign of the actual function is.

b.) Kinetic energy to be negative? If yes, give an everyday example.
Solution: Kinetic energy is a function of mass (something that is never negative) and the magnitude of the velocity *squared* (something else that is never negative). In short, kinetic energy is never negative.

c.) Work quantity to be negative? If yes, give an everyday example.
Solution: If a force opposes the motion of a body, it will do negative work (the angle between the force and displacement will be greater than 90^o, so the cosine part of the dot product will yield a negative number). Though it isn't always the case, sliding friction usually does negative work taking energy out of a system.

d.) Power to be negative? If yes, give an everyday example.
Solution: Power is just *work per unit time*. If work can be negative, so can power. What does negative power mean? It tells you how much energy is being pulled out of the system *per unit time*. Though it probably isn't obvious, the power rating on a light bulb could be a negative number as it tells you how much electrical energy (joules) is converted (i.e., pulled out of the electrical system) to heat and light *per second*. This *joules per second* quantity is called a *watt*.

6.15) The units of power *could* be which of the following (more than one are possible)?

a.) Joules/sec.
Solution: Power is defined as *work per unit time*, so at the very least you would expect *joules/second* to do the job.

b.) Watts/sec.
Solution: *Watts* is the name for the units of power, so *watts/second* isn't a power quantity.

c.) $Kg \cdot m^2/s^3$.

Solution: A *joule* is a $nt \cdot m$ which, in turn, is a $(kg \cdot m/second^2)(m)$, or $(kg \cdot m^2/second^2)$. Dividing this by *seconds* yields a perfectly good representation of power units.

d.) $Nt \cdot m/s$.

Solution: As was pointed out in *Part c*, a *joule/second* is really a $nt \cdot m/s$, so this one works (no pun intended . . . sorta).

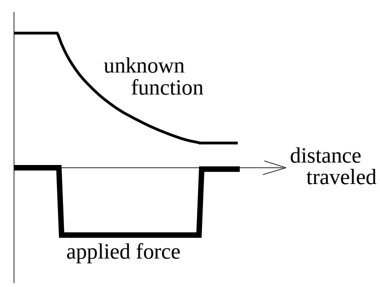
6.16) *Work* is to *energy* as *force* is to *(velocity)*. How so?

Solution: The idea is that doing *work* can potentially alter the *energy* content of a system whereas applying a *force* can potentially alter the *velocity* of a body. This may be obscure, but that's life (it was designed to make you think about *relationships*).

6.17) The potential energy function associated with a spring force of $-kx$ is $.5kx^2$. What would you expect the potential energy function for a force of $-kx^5$ to be? How would you derive such a function?

Solution: The pattern seems to be to raise the exponent by one and create a coefficient equal to the inverse of that number. In other words, the pattern suggests that the function would be $(1/6)kx^6$ (and, in fact, that's what it is). Noting that the force is equal to zero at $x = 0$ (hence, setting the potential energy function equal to zero at that point), the formal derivation of that quantity requires that you solve the expression $U(x) - U(x{=}0) = \int \mathbf{F} \cdot d\mathbf{r}$ evaluated from $x = 0$ to x, where $d\mathbf{r}$ becomes dx after the *dot product* has been done.

6.18) A vehicle moves in the *+x direction*. The net force applied to the vehicle is shown to the right along with a second graph. What might that second graph depict?



Solution: In one dimension, the net force applied to a moving object is directly proportional to the work that force does on the object. In fact, because the displacement is in the $+x$ direction and the force is negative (see the graph), we know that the net work is negative (the force and displacement directions are opposite one another). Net work is related to the change of the body's *kinetic energy*, a quantity that is, itself, related to *velocity squared*. As the unknown quantity seems to be decreasing as long as the force acts, and as the quantity is doing so as a quadratic, my guess is that the unknown function is that of the velocity of the vehicle.

6.19) A force is applied to an object for some period of time t. During that time it does *W's* worth of work. If the time of contact remains the same but the force is doubled, what will the ratio of the work quantities be?

Solution: The temptation is to assume that if the force doubles, the work will double. The problem is that as the acceleration doubles (this follows from doubling the force), the distance traveled will change in time t. In other words, we actually have to do the work calculation for both situations and see how things turn out. Assuming $\boldsymbol{F}$ and $\boldsymbol{d}$ are in the same direction and we start from rest, we can combine the definition of work, N.S.L., and kinematics to write W_1

$= Fd_1 = [ma_1][.5a_1t^2]$ (the substitution for the d_1 term came from the kinematics relationship $d = v_ot + .5at^2$ with $v_o = 0$). Noting that if the force doubles, the acceleration doubles so that $a_2 = 2a_1$, we can write up the new situation as $W_2 = F_2d_2 = [m(2a_1)][.5(2a_1)t^2] = 4[ma_1][.5a_1t^2]$. Taking the ratio of the two work calculations yields 1:4.

6.20) Assume you have a constant force $\mathbf{F} = (12\ newtons)\mathbf{i}$ that does work on a moving object as the object travels a distance $\mathbf{d} = (2\ meters)\mathbf{i}$ in time $t = 3\ seconds$.

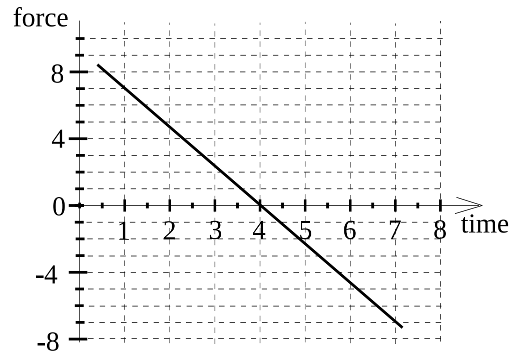
a.) At what rate is energy being pumped into the system?
Solution: The total amount of energy that $\mathbf{F}$ imparts to the object over the distance $\mathbf{d}$ is just equal to the amount of *work* $\mathbf{F}$ does during the trip. The rate of energy flow, then, is that total amount of work done divided by the time it took to do the deed. In other words, the quantity we want is equal to W/t. Using our definition of *work*, this becomes $W/t = \mathbf{F}\cdot\mathbf{d}/t = [(12\ nts)(2\ meters)\cos 0^o]/(3\ seconds) = 8$ *joules/second*.

b.) What is the name given to the quantity you derived in *Part a*?
Solution: A measure of *rate of energy change* (i.e., how quickly energy is being pumped into or out of a system) is called *power*. *Power* can be determined for a single force, on average, over a large period of time (that was the case in *Part a*), for a single force as manifested at a particular point in time (this is *instantaneous* power), or for the *net force* acting on a system. In most cases, you sorta have to look closely to see if you are being asked to determine an average or not. The key to look for is whether the information you are given happens over a long period of time or not (if it is a long period of time, we are talking *average*). In most cases, you will be determining an average.

c.) Come up with four different ways to express the quantity named in *Part b*.
Solution: The cheap response is to write W/t (this is really weak, but it's also correct). As *work* is defined as $\mathbf{F}\cdot\mathbf{d}$, this can be expanded yielding $W/t = \mathbf{F}\cdot\mathbf{d}/t = (Fd\cos\theta)/t$. Note that you are really determining the *average power* over the time t during which the object moves a distance d. Another possibility is to note that an object that travels a distance d in time t has some average velocity (call it $\mathbf{v}$). Manipulating, we can rewrite our original expressions as $W/t = \mathbf{F}\cdot\mathbf{d}/t = \mathbf{F}\cdot\mathbf{v}$. This little gem is nice because it relates the power provided by a constant force $\mathbf{F}$ to an object moving with constant velocity $\mathbf{v}$. If the force happens to be changing, the power function simply becomes a function of time. The most exotic of the expressions for power (exotic in the sense that you don't see it used much) comes from the *work/energy* theorem. Remembering that the net work provided to a body equals the body's *change of kinetic energy*, we can write $W_{net}/t = \Delta(.5mv^2)/t$. This obviously refers to the total power being provided to a system, not the power associated with a particular force in the system. In any case, there are your four ways. Note: When you are doing *power* problems, remember what you are dealing with. *Power* is the measure of the rate at which energy is put into or taken out of a system.

d.) In the MKS system, what are the units for this quantity and what are the units called?

Solution: The units for *work per unit time* are *joules per second*. In the MKS system, this quantity is called a *watt*.



6.21) The graph shows the force $\boldsymbol{F}$ applied to an object that moves with a *constant velocity* of *.5 m/s* in the *-**i** direction*. Assuming $\boldsymbol{F}$ is oriented along the *x axis*:

a.) What can you say about the other forces that act in the system?

Solution: If $\boldsymbol{F}$ is changing but the velocity is constant, the other forces in the system must vectorially add to $-\boldsymbol{F}$. In that way, the net force will be zero and the change of kinetic energy (work/energy theorem) will be zero.

b.) How much power does $\boldsymbol{F}$ provide to the object between $t = 1$ *second* and $t = 7$ *seconds*?

Solution: With the velocity constant, the body moves in one direction (the *-**i** direction*) and one direction only. While the force is in the positive direction, therefore, it provides negative work to the body. While in the negative direction, it provides positive work to the body. Between $t = 1$ *second* and $t = 7$ *seconds*, it spends as much time doing the one as the other. As such, it makes sense that its average work will be zero during that interval and, hence, the average power from $\boldsymbol{F}$ will also be zero. Put a little differently, whatever energy $\boldsymbol{F}$ takes out of the system between $t = 1$ *second* and $t = 4$ *seconds*, it puts back into the system between $t = 4$ *seconds* and $t = 7$ *seconds*.

c.) After $t = 4$ *seconds*, $\boldsymbol{F}$*'s* direction changes. What does that say about the power associated with $\boldsymbol{F}$ from then on?

Solution: Given the fact that the direction of motion doesn't change at $t = 4$ *seconds*, $\boldsymbol{F}$ will switch from taking energy *out of* the system to putting energy *into* the system at that point. Note that just because $\boldsymbol{F}$ is negative after $t = 4$ *seconds* doesn't mean it is taking energy from the system. The WORK $\boldsymbol{F}$ does determine what the power is doing. After $t = 4$ *seconds*, the work done by $\boldsymbol{F}$ is positive. As a consequence, energy is being transferred *into* the system by $\boldsymbol{F}$ from then on.

d.) How much power, on average, does $\boldsymbol{F}$ provide between $t = 1$ *second* and $t = 4$ *seconds*?

Solution: There are all sorts of ways we can do this. Noting that the magnitude of the average force over that interval is $\Delta F/2 = (7\ nt)/(2) = 3.5\ joules$, we could write $P_{avg} = \boldsymbol{F}_{avg} \cdot \boldsymbol{v} = (3.5\ j)(.5\ m/s)\cos 180^o = -1.75\ watts$. Another possibility would be to note that with a constant velocity of .5 m/s, the body would travel 1.5 meters in three seconds. With that we could write $P_{avg} = \boldsymbol{F}_{avg} \cdot \boldsymbol{d}/t = [(3.5\ j)(1.5\ m)\cos 180^o]/(3\ sec) = -1.75\ watts$.

e.) As an interesting twist, given that the average power provided to the system between $t = 4$ *seconds* and $t = 7$ *seconds* is +1.75 watts, how much work does the force do during that period of time?

Solution: If $P_{avg} = W/t$ for the interval, then $W = P_{avg}t = (1.75\ j/s)(3\ s) = 5.25\ joules$ of work.

6.22) Let's assume that a car engine provides a constant amount of power. The car accelerates from zero to 30 m/s. Is the car's acceleration constant?

Solution: The power provided by a net force $\boldsymbol{F}$ being applied to a car traveling at a given velocity is $\boldsymbol{F}\cdot\boldsymbol{v}$. If we assume the force is in the direction of motion (a good assumption if the car's speed is increasing), how does constant power affect this relationship? Using N.S.L., we can rewrite the magnitude of the force as ma, where a is the acceleration of the vehicle and m is its mass. Doing the *dot product*, then substituting $m\boldsymbol{a}$ in for $\boldsymbol{F}$ yields a power relationship that states $\boldsymbol{F}\cdot\boldsymbol{v} = (ma)v = constant$. The only way the power can *remain* constant as the velocity increases is if the acceleration decreases. This isn't too weird. It takes a lot more energy to increase the kinetic energy of a body some amount at high velocity than it does to increase the kinetic energy that same amount at low velocity. If the *rate of energy transfer* is the same throughout, one would expect the velocity change (acceleration) to happen more slowly as the velocity gets bigger.

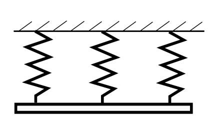
6.23) A group of students were asked the following question: "In the real world, what does the power requirement do as you double a car's velocity?" Assuming a reasonable answer was expected, what information *is missing* in the set-up? That is, what additional information would the students have needed to answer sensibly?

Solution: In cars, power output tells us how many *joules per second* (or *horsepower* as the case may be) must be burned to keep the car moving at a given speed. There are several ways to express this power relationship: W/t or $\boldsymbol{F}\cdot\boldsymbol{d}/t$ or $\boldsymbol{F}\cdot\boldsymbol{v}$ or $\Delta(.5mv^2)/\Delta t$ (it should be evident to you how each of these relationships follows from one another). The $\boldsymbol{F}\cdot\boldsymbol{v}$ expression might nudge people into believing that doubling the velocity would require a doubling of the power requirement. The problem is that in the real world, it takes more force to maintain a higher velocity (a car traveling at higher velocity will have more road and air friction to contend with). How much more force? That's the problem. We don't know.

6.24) In his younger days, George boasted he could do a million joules of work. Gertrude, his betrothed, wasn't impressed. Why do you suppose she wasn't moved?

Solution: A million joules of work is impressive, but not if you take an entire lifetime to do it. *Power* yields the amount of work done *per unit time*. George's *power rating* is what Gertrude should be interested in.

6.25) Three identical springs are attached at the ceiling. A bar of mass m is hooked to the group. If the new system's equilibrium position is d units below the springs' unstretched lengths, what



must the spring constant be for each spring? Use *energy considerations* to dismantle this problem.

Solution: This was *Problem 27* in the N.S.L. chapter. This time we are going to look at it from the perspective of energy. What makes this problem interesting is that if you try to mindlessly plug and chug, you will get the wrong answer. That is, if you say that the spring potential energy when the bar is at the new equilibrium position is equal to the gravitational potential energy when the bar was at the unextended position, you will write $3[(1/2)kd^2] = mgd$, or $k = (2/3)mg/k$. This isn't what we got when we untangled the situation using N.S.L., so we have a problem.

What you need to realize is that in this case, you have to set up a single problem, then think conceptually. The problem I'd consider is as follows.

Assume the bar starts out d units above the new equilibrium position (i.e., in the position the bar was in as it was being hooked to the springs). From that position, assume the bar is released. The bar will accelerate down toward the new equilibrium position d units below the start point, move through that the new equilibrium position continuing on until it gets d units below the new equilibrium position, then begin back up toward the new equilibrium position. In other words, it will oscillate with amplitude d about the new equilibrium position.

For that scenario, how much energy do the *springs* have at the start point?

They have none as they are not elongated at all.

How much gravitational potential energy will the bar have at the start point?

Taking the new equilibrium position to be the *zero gravitional potential energy point*, there will be *mgd's* worth.

All of this means we start out with *mgd's* worth of potential energy in the system.

Once the bar has fallen to the new equilibrium position, the bar has zero gravitational potential energy (that was where we defined the zero point to be for gravitational PE), the springs will each have $(1/2)kd^2$'s worth of potential energy, and the bar will have KINETIC ENERGY in the amount $(1/2)mv^2$, where v is the maximum velocity the bar will ever have (the velocity of any oscillating object is always maximum when it moves through its equilibrium position).

The maximum velocity of a vibrating system is equal to ωA, where $\omega = (k_{net}/m)^{1/2}$ and the amplitude of the motion in this case is $A = d$.

The net, effective spring constant of three identical springs "in parallel," so to speak, is $k_{net} = 3k$, (remember, k is the spring constant for each spring--this is what we are looking for).

Putting everything together, we can write $v = \omega A = (3k/m)^{1/2}d$, and with this we can write the conservation energy expression as:

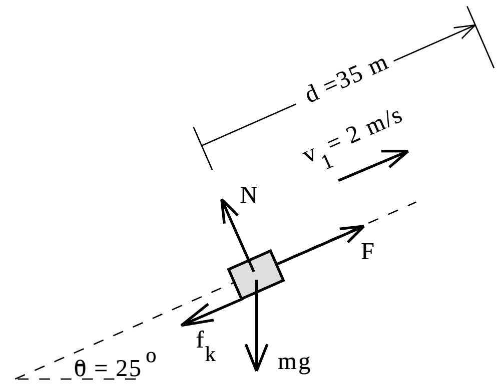
$$mgd = (1/2)m[(3k/m)^{1/2}d]^2 + 3[(1/2)kd^2].$$

Solving for k yields (lo and behold) $k = mg/3d$, the very answer we got in the N.S.L. chapter.

The reason this problem was included, and the moral of the story, is that simply because things like *potential energy zero levels* have generally accepted working definitions, there are time when you have to use your head above and beyond the classical definitions. In those cases, you have to be conceptually thoughtful and just a little bit clever.

PROBLEM SOLUTIONS

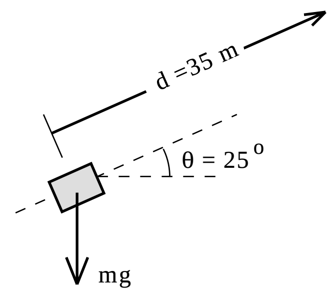
6.26) The f.b.d. shown to the right has been provided to identify all the forces acting on the body as it moves up the incline.



a.) To determine the work done by gravity as the body moves up the incline, there are two approaches. For your convenience, the force, velocity, and displacement are pictured below and also to the right.

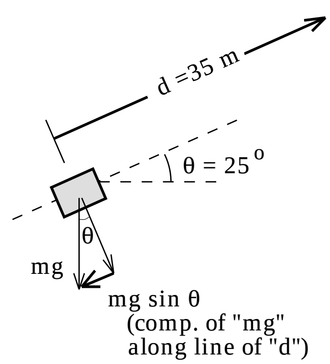
Approach #1: Using the definition of work and the angle between *mg* and *d* as ϕ:

$$
\begin{aligned}
W_{grav} &= \mathbf{F}_{grav} \cdot \mathbf{d} \\
&= (mg)\,(d)\cos\phi \\
&= (mg)\,(d)\cos(25^o + 90^o) \\
&= (3\text{ kg})(9.8\text{ m/s}^2)(35\text{ m})\,(-.423) \\
&= -434.87\text{ joules.}
\end{aligned}
$$



Approach #2: Using the component of *mg* along the line of *d*:

$$
\begin{aligned}
W_{grav} &= \mathbf{F}_{grav} \cdot \mathbf{d} \\
&= \underline{+}(F_{\text{mg parallel to "d"}})\,(d) \\
&= -(mg\sin 25^o)\,(d) \\
&= -[(3\text{ kg})(9.8\text{ m/s}^2)(.423)]\,(35\text{ m}) \\
&= -434.87\text{ joules.}
\end{aligned}
$$

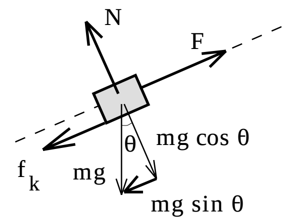


Note: In this case, the force is OPPOSITE the direction of the displacement which means the work must be negative. The *negative sign* in this case must be inserted *manually*. An alternative would be to notice that the angle ϕ between *d* and *F's* component along *d's* line is 180^o and determine the work quantity using:

$$W_{grav} = \mathbf{F}_{grav} \cdot \mathbf{d}$$
$$= (F_{\text{mg parallel to "d"}})\ (d) \cos\ \phi$$
$$= (mg \sin 25^o)\ (d) \cos 180^o.$$

In doing so, the cosine function will give you the *-1* automatically.

b.) The frictional force is equal to $\mu_k N$. To determine *N*, we need to use an f.b.d. and N.S.L. in the *normal direction*. The f.b.d. is shown to the right. N.S.L. yields:



$\underline{\sum F_N:}$

$$N - mg \cos\ \theta = 0 \quad (\text{as } a_N = 0)$$
$$\Rightarrow \quad N = mg \cos\ \theta$$
$$\Rightarrow \quad f_k = \mu_k N$$
$$= \mu_k (mg \cos\ \theta)$$
$$= (.3)\ (3\ kg)(9.8\ m/s^2) \cos 25^o$$
$$= 7.99\ nts.$$

Friction is always opposite the direction of motion. The *work* friction does will be:

$$W_f = \mathbf{f}_k \cdot \mathbf{d}$$
$$= (f_k)\ (d) \cos 180^o$$
$$= -\ f_k d$$
$$= -(7.99\ nts)(35\ m)$$
$$= -279.65\ joules.$$

c.) The angle between *d* and *N* is 90^o. The cosine of 90^o is zero. That means that the *net work* done by the *normal force* will be zero . . . ALWAYS!

d.) Kinetic energy is defined as *(1/2)mv^2*. Using that expression we get:

$$KE_1 = (1/2)mv^2$$
$$= .5\ (3\ kg)\ (2\ m/s)^2$$
$$= 6\ joules.$$

e.) The work/energy theorem states:

$$W_{net} = \Delta KE.$$

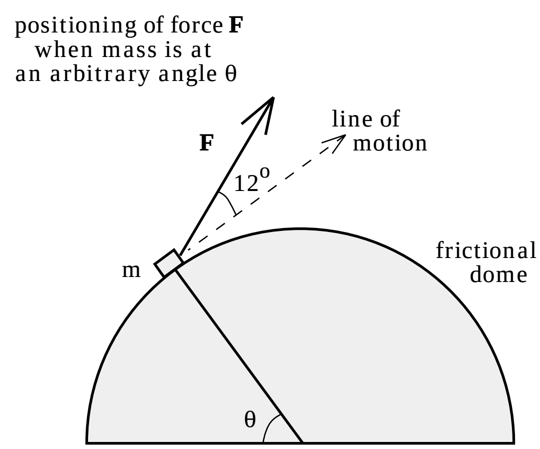
For this case, that means:

$$
\begin{aligned}
W_{net} \quad &= \quad \Delta KE. \\
W_f \quad + \quad W_F \quad + \quad W_{mg} \quad &= \quad KE_2 \quad - \quad KE_1. \\
(f_k)\ (d) \cos 180^o + F\ (d) \cos 0^o + (mg)\ (d) \cos \phi &= (1/2)mv_2{}^2 - (1/2)mv_1{}^2.
\end{aligned}
$$

Plugging in the numbers, we get:

$$(-279.65 \text{ J}) \quad + F(35 \text{ m}) + (-434.87 \text{ J}) = (1/2)(3 \text{ kg})(7 \text{ m/s})^2 - (1/2)(3 \text{ kg})(2 \text{ m/s})^2$$
$$\Rightarrow \quad F = 22.34 \text{ nts.}$$

6.27) The situation is shown in the sketch to the right. The arc length distance *s* the mass moves through as it proceeds from 20^o to 60^o is given by the relationship $s = R\Delta\theta$, where $\Delta\theta$ has to be in radians. Using that yields



$$
\begin{aligned}
s &= R\Delta\theta \\
&= (.3 \text{ meters})\left[\frac{2\pi \text{ radians}}{360^o}\right](60^o - 20^o) \\
&= .21 \text{ meters.}
\end{aligned}
$$

With this information and a constant angle between ***F*** and ***d***, we can write

$$
\begin{aligned}
W &= F \bullet d \\
&= |F||d| \ \cos\phi \\
&= \left(\frac{mg}{4}\right)(.21) \ \cos 12^o \\
&= \left(\frac{(.5 \text{ kg})(9.8 \text{ m/s}^2)}{4}\right)(.21) \ \cos 12^o \\
&= .252 \text{ joules.}
\end{aligned}
$$

6.28) All the energy is stored in spring potential energy. Using the potential energy function for a spring we get:

$$\begin{aligned} U_{sp} &= (1/2)kx^2 \\ &= .5(120\text{ nt/m})(.2\text{ m})^2 \\ &= 2.4\text{ joules.} \end{aligned}$$

6.29)

a.) We could use the *work/energy theorem* on this problem, but the *modified conservation of energy equation* is so much easier to use that we will use it here. Noting that the tension in the line is always *perpendicular to the motion* (i.e., the work done due to tension is zero), we can write:

$$\begin{array}{ccccccccc} KE_1 & + & \sum U_1 & + & \sum W_{extra} & = & KE_2 & + & \sum U_2 \\ (1/2)mv_1^2 & + & mgy_1 & + & Td\cos 90^o & = & (1/2)mv_2^2 & + & mgy_2 \\ (0) & + & m(9.8\text{ m/s}^2)(12\text{ m}) & + & (0) & = & (1/2)mv_2^2 & + & m(9.8\text{ m/s}^2)(5\text{ m}). \end{array}$$

Being careful not to confuse mass terms denoted by *m* and the units of length (meters, abbreviated *m*), we can cancel the mass terms and get:

$$v_2 = 11.71\text{ m/s.}$$

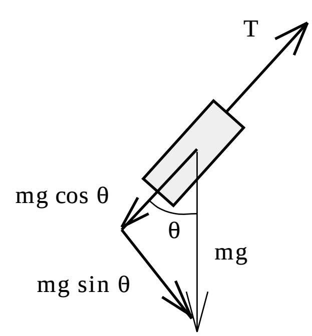
b.) At the bottom of the arc, Tarzan's velocity can again be found using the *modified conservation of energy* expression (we need that velocity because he is moving under the influence of a center-seeking force--a velocity driven function--at that point). Using the approach:

$$\begin{array}{ccccccccc} KE_1 & + & \sum U_1 & + & \sum W_{extra} & = & KE_{bot} & + & \sum U_{bot} \\ (1/2)mv_1^2 & + & mgy_1 & + & (0) & = & (1/2)mv_{bot}^2 & + & mgy_2 \\ (0) & + & m(9.8\text{ m/s}^2)(12\text{ m}) & + & (0) & = & (1/2)mv_{bot}^2 & + & m(9.8\text{ m/s}^2)(2\text{ m}). \end{array}$$

Canceling the mass terms yields:

$$v_{bot} = 14\text{ m/s.}$$

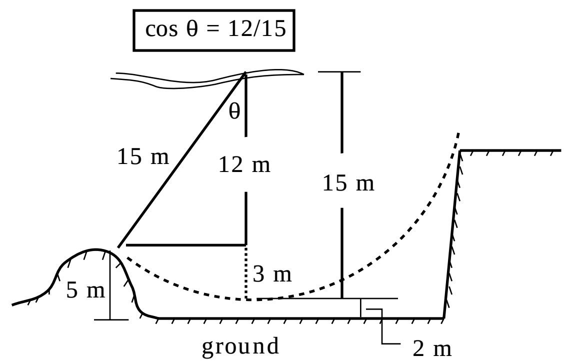
There is a *tension force* upward and *gravity* downward when Tarzan is at the bottom of the arc (you might want to draw an f.b.d. to be complete). N.S.L. suggests:



$\underline{\sum F_c:}$

$$T - mg = ma_c$$
$$= m(v^2/R) \qquad \text{(as } a_c \text{ is a centripetal acc.)}$$
$$\Rightarrow \quad T = mg + mv^2/R$$
$$= (80\text{ kg})(9.8\text{ m/s}^2) + (80\text{ kg})(14\text{ m/s})^2/(15\text{ m})$$
$$= 1829.3\text{ nts.}$$

c.) At the molehill, Tarzan's velocity is 11.71 m/s. An f.b.d. for that situation is shown. The only thing that is really tricky about the problem is determining the angle θ. The diagram below-right will do that.



With θ, N.S.L. yields:

$\underline{\sum F_c:}$

$$T - mg\cos\theta = ma_c$$
$$= m(v^2/R).$$

As a_c is a centripetal acceleration:

$$\Rightarrow \quad T = mg\cos\theta + mv^2/R$$
$$= (80\text{ kg})(9.8\text{ m/s}^2)(12/15) + (80\text{ kg})(11.71\text{ m/s})^2/(15\text{ m})$$
$$= 1358.5\text{ nts.}$$

6.30) Both gravity and friction do *work* as the body slides down the incline. Using the *modified conservation of energy equation*, we get:

$$KE_1 + \sum U_1 + \sum W_{extra} = KE_2 + \sum U_2$$
$$(1/2)mv_1^{\,2} + mgy_1 + (-f_k d) = (1/2)mv_2^{\,2} + mgy_2$$
$$(0) + mgR - (f_k d) = (0) + (0)$$
$$\Rightarrow \quad f_k = mgR/d.$$

In this case, d is the total distance over which the frictional force acts. That is, the 18 meters along the horizontal surface AND the quarter circumference down the curved incline (that will equal $(1/4)(2\pi R)$). That total distance is:

$$\begin{aligned} d &= 18 + .5\pi R \\ &= 18 + .5\pi(2 \text{ m}) \\ &= 21.14 \text{ m}. \end{aligned}$$

Plugging this into our expression, we get:

$$\begin{aligned} f_k &= mgR/d \\ &= (12 \text{ kg})(9.8 \text{ m/s}^2)(2 \text{ m})/(21.14) \\ &= 11.13 \text{ nts}. \end{aligned}$$

6.31) The fact that the angle is 85^o makes no difference, assuming the velocity is great enough to allow the dart to make it to the monkey. What is important is that the dart has enough energy in the beginning to effect a pierce at the end. Using *conservation of energy*:

$$\begin{array}{ccccccccc} \sum KE_1 & + & \sum U_1 & + & \sum W_{ext} & = & \sum KE_2 & + & \sum U_2 \\ (1/2)mv_1{}^2 & + & (0) & + & (0) & = & (1/2)mv_2{}^2 & + & mgh_2. \end{array}$$

Dividing out the masses and multiplying by 2 yields:

$$\begin{aligned} v_1 &= [v_2{}^2 + 2gh_2]^{1/2} \\ &= [(4 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(35 \text{ m})]^{1/2} \\ &= 26.5 \text{ m/s}. \end{aligned}$$

6.32) Assuming an average frictional force of 27 newtons:

a.) Let the *potential energy equals zero* level be the ground. That means that below-ground level, the h in mgh will be *negative*. Using *conservation of energy*:

$$\begin{array}{ccccccccc} KE_1 & + & \sum U_1 & + & \sum W_{ext} & = & KE_2 & + & \sum U_2 \\ (1/2)mv_1{}^2 & + & (0) & + & (-f_k d) & = & (1/2)mv_C{}^2 & + & mgh_C. \end{array}$$

or

$$.5(1800\text{ kg})(38\text{ m/s})^2 - (27\text{ nts})(130\text{ m}) = .5(1800\text{ kg})v_C^{\,2} + (1800\text{ kg})(9.8\text{ m/s}^2)(-15\text{ m})$$
$$\Rightarrow \quad v_C = 41.64\text{ m/s.}$$

b.) The only thing that is tricky about this is finding the total distance *d* traveled (we need that to determine the amount of work friction does). Noting that the distance traveled while moving through the loop is $2\pi R$, *conservation of energy* yields:

$$\begin{array}{ccccccc} KE_1 & + \ \Sigma U_1 & + & \Sigma W_{ext} & = KE_2 + & \Sigma U_2 \\ (1/2)mv_1^{\,2} + & (0) & + & (-f_k d_{tot}) & = \ (0) & + \ mgh_{ramp} \\ (1/2)mv_1^{\,2} + & 0 & & -\ f_k(70+60+40+2\pi(20)+d) & = \ \ 0 & + \ mgd\sin\theta \end{array}$$
$$.5(1800\text{ kg})(38\text{ m/s})^2 - [(27\text{ nts})(295.7\text{ m}) + 27d] = (1800)(9.8\text{ m/s}^2)d\sin 30^\circ$$
$$\Rightarrow \quad d = 146\text{ m.}$$

c.) Along with the *conservation of energy*, this problem requires N.S.L. We know that at the top of the arc, the *vertical forces* are centripetal at the velocity required AND the *normal force* goes to zero. From the *conservation of energy* we get:

$$\begin{array}{cccccc} KE_1 & + \ \Sigma U_1 + & \Sigma W_{ext} & = & KE_2 & + & \Sigma U_2 \\ (1/2)mv_1^{\,2} + & (0) \ + & (-f_k d_{top}) & = & (1/2)mv_{top}^{\,2} & + & mgh_{top} \\ (1/2)mv_1^{\,2} \ + & 0 & -\ f_k[70+60+40+(2\pi R)/2] & = & (1/2)mv_{top}^{\,2} & + & mg(2R) \end{array}$$
$$.5(1800\text{ kg})v_1^{\,2} + \ 0 \ - (27\text{ nts})(232.8\text{ m}) = .5(1800\text{ kg})v_{top}^{\,2} + (1800)(9.8\text{ m/s}^2)[2(20\text{ m})]$$
$$\Rightarrow \quad v_1^{\,2} = v_{top}^{\,2} + 791.$$

Using N.S.L., we get:

$$\begin{aligned} -N - mg &= -ma_c \\ &= -m(v_{top}^{\,2}/R). \end{aligned}$$

When the velocity is correct, *N* goes to zero and:

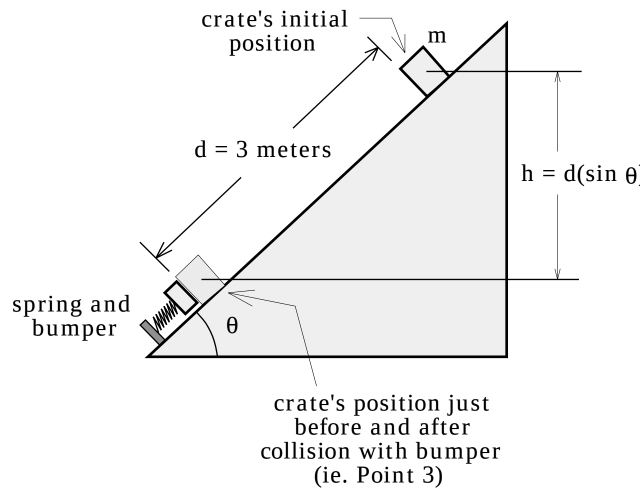
$$v_{top}^{\,2} = gR.$$

Substituting that into our expression, we get:

$$v_1^{\,2} = v_{top}^{\,2} + 791$$
$$= gR + 791$$
$$= (9.8 \text{ m/s}^2)(20 \text{ m}) + 791$$
$$\Rightarrow \quad v_1 = 31.42 \text{ m/s}.$$

6.33)

a.) Assuming the spring is depressed a distance *x*, let's define the *gravitational potential energy equals zero* level to be at that point (i.e., where the spring is depressed a distance *x*). As such, the crate travels a distance $d + x$, where d is the crate's initial distance from the bumper. The *modified conservation of energy theorem* implies:



$$KE_1 + \sum U_1 + \sum W_{ext} = KE_2 + \sum U_2$$
$$(0) + [\,U_{grav,1} + U_{sp,1}\,] + \sum W_{ext} = (0) + [\,U_{grav,2} + U_{sp,2}\,]$$
$$(0) + [mg(d+x)\sin\theta + (0)\,] - f_k(d+x) = (0) + [\,(0) + .5kx^2\,]$$
$$\Rightarrow \quad mg(d+x)\sin\theta - f_k(d+x) = .5kx^2$$
$$\Rightarrow \quad (60 \text{ kg})(9.8 \text{ m/s}^2)(3 + x)\sin 55^o - (100 \text{ nt})(3 + x) = .5(20{,}000)x^2.$$

Rearranging:

$$10{,}000x^2 - 381.7x - 1145 = 0.$$

Using the Quadratic Formula, we get *x = .36 meters*.

b.) In this section, we really are not interested in *x*--we want to know how much energy the block has just before hitting the spring, and how much energy the block loses by the time it leaves the spring. In other words, this is really a brand new problem. As such, let's redefine the *zero gravitational potential energy level* to be at the point when the block is just about to come into contact with the spring. If that be the case, the *conservation of energy* allows us to determine the energy of the block just before striking the spring at *Point 3*:

$$KE_1 + \Sigma U_1 + \Sigma W_{ext} = KE_3 + \Sigma U_3$$
$$(0) + mg\,d\sin\theta - f_k d = KE_3 + (0)$$

$$\Rightarrow \quad KE_3 = mgd\sin\theta - f_k d$$
$$= (60\text{ kg})(9.8\text{ m/s}^2)(3\sin 55^o) - (100\text{ nt})(3\text{ m})$$
$$= 1144.98\text{ joules.}$$

If 3/4 of the kinetic energy is lost, then *1/4* is left. That means the block has kinetic energy *(1/4)(1144.98 j) = 286.2 joules* as it starts back up the incline. If we let *L* be the distance the block travels up the incline to rest, and if we remember that the *U = 0 level* is at the spring's end, we have:

$$KE_4 + \Sigma U_4 + \Sigma W_{ext} = KE_5 + \Sigma U_5$$
$$(286.2\text{ j}) + (0) - (100\text{ nt})L = (0) + (60\text{ kg})(9.8\text{ m/s}^2)(L\sin 55^o)$$
$$\Rightarrow \quad L = .49\text{ meters.}$$

6.34) Using the potential energy function provided, *conservation of energy* implies:

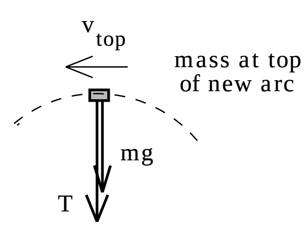
$$KE_1 + \Sigma U_1 + \Sigma W_{ext} = KE_2 + \Sigma U_2$$
$$(1/2)m_s v_1^2 + [-Gm_e m_s/(r_e + d_1)] + (0) = (1/2)m_s v_2^2 + [-Gm_e m_s/(r_e + d_2)].$$

Noticing that the m_s*'s* cancel out, we can write:

$$.5(1500\text{ m/s})^2 + [-(6.67\text{x}10^{-11}\text{ m}^3/\text{kg}\cdot\text{s}^2)(5.98\text{x}10^{24}\text{ kg})/(6.37\text{x}10^6\text{ m} + 1.2\text{x}10^5\text{ m})] =$$
$$.5v_2^2 + [-(6.67\text{x}10^{-11}\text{ m}^3/\text{kg}\cdot\text{s}^2)(5.98\text{x}10^{24}\text{ kg})/(6.37\text{x}10^6\text{ m} + .9\text{x}10^5\text{ m})].$$

SOLVING yields $v_2 = 1680\text{ m/s}$.

6.35) The trick to this problem is in recognizing the fact that at the top of its arc, the mass is executing *centripetal acceleration* where *tension* is acting as one of the *centripetal forces* in the system. Using N.S.L. and remembering that the radius of the mass's motion is *L/3*, we get:

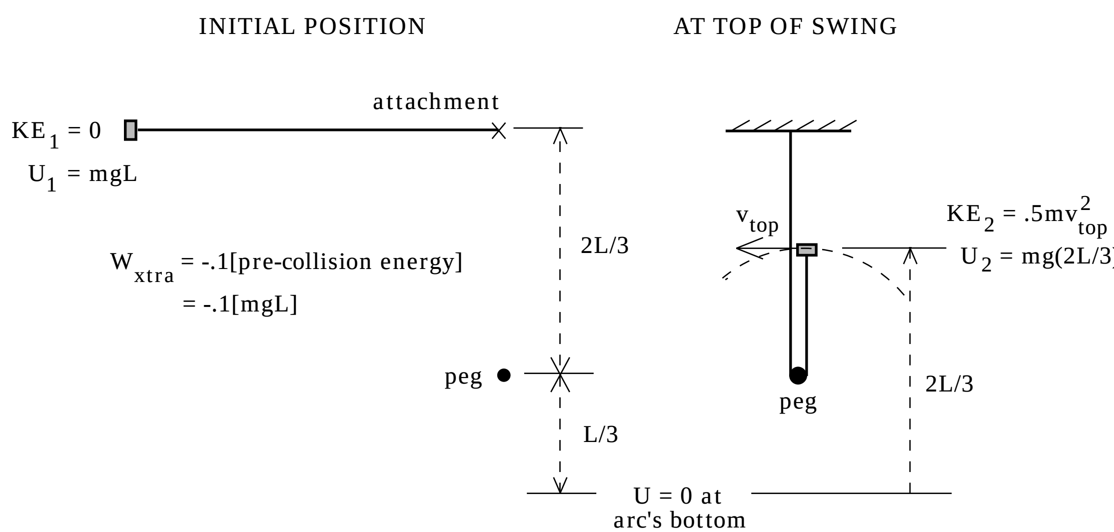


$\underline{\sum F_c}$:

$$
\begin{aligned}
-T - mg &= -ma_c \\
&= -m(v_{top}{}^2/R) \\
&= -m(v_{top}{}^2/(L/3)) \\
\Rightarrow \quad T &= (3m/L)(v_{top})^2 - mg.
\end{aligned}
$$

To solve this expression for *T*, we need the velocity of the mass at the top of its flight.

Enter the *modified conservation of energy* approach--an approach designed specifically to determine velocities when conservative force fields are doing work on bodies. If we take the *potential energy equal to ZERO* point to be at the bottom of the arc, the information given in the sketch tells it all.



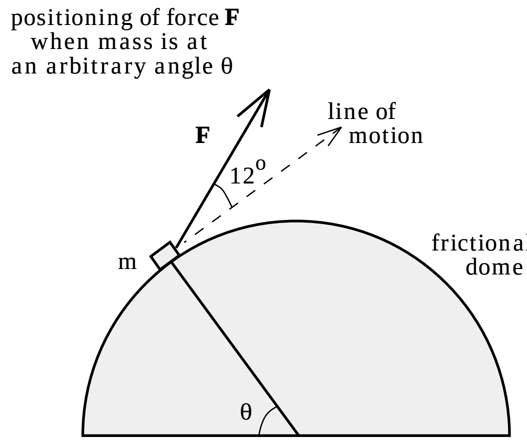
Using our information, we get:

$$
\begin{array}{ccccccccc}
KE_1 & + & \sum U_1 & + & \sum W_{ext} & = & KE_2 & + & \sum U_2 \\
(0) & + & mgL & + & [-(.1)(mgL)] & = & (1/2)mv_{top}{}^2 & + & mg(2L/3)
\end{array}
$$

$$\Rightarrow \quad v_{top} = [.46gL]^{1/2}.$$

Plugging this into our *tension* expression yields:

$$\begin{aligned} T &= (3m/L)(v_{top})^2 - mg \\ &= (3m/L)([.46gL]^{1/2})^2 - mg \\ &= 1.38mg - mg \\ &= .38mg. \end{aligned}$$



6.36) The mass begins from rest and is motivated up the incline by the force *F*. On the way up the incline, gravity, friction, and the force ***F*** all do work on the body. *Conservation of energy* is the ideal approach for this. Remembering that the arc length *d* associated with the mass's motion is .21 meters up the incline (we determined that in *Problem 2*), the average normal force was .4mg, the coefficient of kinetic friction was .6, and the force ***F*** was always 12^o off the line of motion, we can write the following.

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + mg(d_1) + (W_F + W_f) = \left(\tfrac{1}{2}\right)mv_2^{\,2} + (mgd_2)$$

$$mg(R\sin 20^o) + \left(|\mathbf{F}|\,|\mathbf{d}|\cos 12^o + \left(|\mathbf{f}|\,|\mathbf{d}|\cos 180^o\right)\right) = \left(\tfrac{1}{2}\right)mv_2^{\,2} + mg(R\sin 60^o)$$

$$mg(R\sin 20^o) + \left(\left(\frac{mg}{4}\right)d\cos 12^o + \left((\mu_k\ N)\ d\right)(-1)\right) = \left(\tfrac{1}{2}\right)mv_2^{\,2} + mg(R\sin 60^o)$$

$$mg(R\sin 20^o) + \left(\left(\frac{mg}{4}\right)d\cos 12^o + \left(-(\mu_k(.4mg))d\right)\right) = \left(\tfrac{1}{2}\right)mv_2^{\,2} + mg(R\sin 60^o)$$

Ignoring units to conserve space, this comes out to be:

$$m\ g\ (R\sin 20^o) + \left(\left(\frac{mg}{4}\right)d\ \cos 12^o + \left(-(\mu_k(.4\ m\ g))\ d\right)\right) = \left(\tfrac{1}{2}\right) m\ v_2^{\,2} + m\ g\ (R\sin 60^o)$$

$$(.5)(9.8)(.3\sin 20^o) + \left(\left(\frac{(.5)(9.8)}{4}\right)(.21)\cos 12^o - (.6)(.4\ (.5)(9.8))(.21)\right) = \left(\tfrac{1}{2}\right)(.5)v_2^{\,2} + (.5)(9.8)(.3\sin 60^o)$$

$$\Rightarrow \quad v_2 = (.283)^{1/2}$$

$$\Rightarrow \quad v_2 = .53\ m/s.$$

Note: <u>On a test</u>, the only lines you would need provide would be lines 1, 2, 5, 7 and 9.